

## 7.3 SAMPLE MEANS FROM NORMAL POPULATIONS

### THE MEAN AND STANDARD DEVIATION OF $\bar{x}$

Thus far, we have studied sampling distributions in general and the sampling distribution of a sample proportion in particular. Now we'll turn our attention to the sampling distribution of another statistic: the sample mean  $\bar{x}$ . Note that we have already looked at one example of the sampling distribution of the sample mean: the sample mean *total gross* for top movies.

1. Go to the website [http://onlinestatbook.com/stat\\_sim/sampling\\_dist/](http://onlinestatbook.com/stat_sim/sampling_dist/). Click the “Begin” button on the left side. Notice the population at the top of the page. Record the values of the mean and standard deviation in the top of the table below. Also, record the shape of the population.

	Population Mean $\mu =$	Population SD $\sigma =$	Population Shape:
Sample Size	Mean of $\bar{x}$ distribution	Std. Dev. of $\bar{x}$ distribution	Shape of $\bar{x}$ distribution
n = 5			
n = 10			
n = 20			
n = 25			

2. Click the “Animated” button and watch a sample of size 5 be drawn from the population. Notice that the sample mean is calculated and plotted on its own number line. Click “Animated” a few more times, watching the animation. Did you obtain the same sample each time? Did you obtain the same sample mean each time?
3. Click the button labeled “5”. This stops the animation for the sample and just calculates and plots another five sample means. Now click the “5” button a few more times. Do you notice a pattern starting to form in the sample means?

4. It's difficult to see patterns without lots of repetitions. Click the "10,000" button. This will generate 10,000 sample means, each one from a different sample of size 5 individuals from the population. Record the mean, standard deviation, and shape of the sample means in the first row of the table on the front page.
  
5. Now click "Clear lower 3." Click on the top "N=5" button and select "N=10" from the dropdown menu. Click "Animated." How is this simulation different than the first simulation?
  
  
  
  
  
  
  
  
  
  
6. Click on the "10,000" button. Record the mean, standard deviation, and shape of the sample means in the second row of the table on the front page.
  
  
  
  
  
  
  
  
  
  
7. Now click "Clear lower 3." To speed up this process, set up the two bottom number lines so that the top one (third down) is for samples of size "N=20" and the bottom one (fourth down) is for samples of size "N=25." Click "Animated" to make sure that the two number lines are set up correctly. Then, click the "10,000" button. Record the mean, standard deviation, and shape of the sample means in the corresponding rows of the table on the front page.
  
  
  
  
  
  
  
  
  
  
8. Look at the table on the front of this handout and write down any patterns you see.

9. Is the sample mean a biased or unbiased estimator of the population mean? How can you tell?

10. Complete the statement “As the sample size increases, \_\_\_\_\_.” (be very specific in your answer)

11. What shape does the sampling distribution of  $\bar{x}$  have when sampling from a Normal population?

Here is a summary of what we have seen so far:

- The **mean of the sample means** is the population mean. In symbols,  $\mu_{\bar{x}} = \mu$ .
- The **standard deviation of the sample means** decreases as  $n$  increases. In fact, there is a formula to compute the standard deviation of the sample mean when sampling with replacement:  $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ . When sampling without replacement, this formula is approximately correct whenever the sample size is less than 10% of the population size.
- The **shape of the sample means** is Normal when sampling from a Normal population.

12. Check the formula for standard deviation of  $\bar{x}$  for any two of the sample sizes from your table. Do they agree reasonably well with the values from the simulation?

## SOLVING PROBLEMS WITH THE SAMPLING DISTRIBUTION OF $\bar{x}$

13. Turn to p.462 and complete Exercise 50 in your textbook.

14. Turn to p.462 and complete Exercise 52 in your textbook.

15. Turn to p.462 and complete Exercise 54 in your textbook.